Quantum friction on atoms after acceleration

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Outline of this talk

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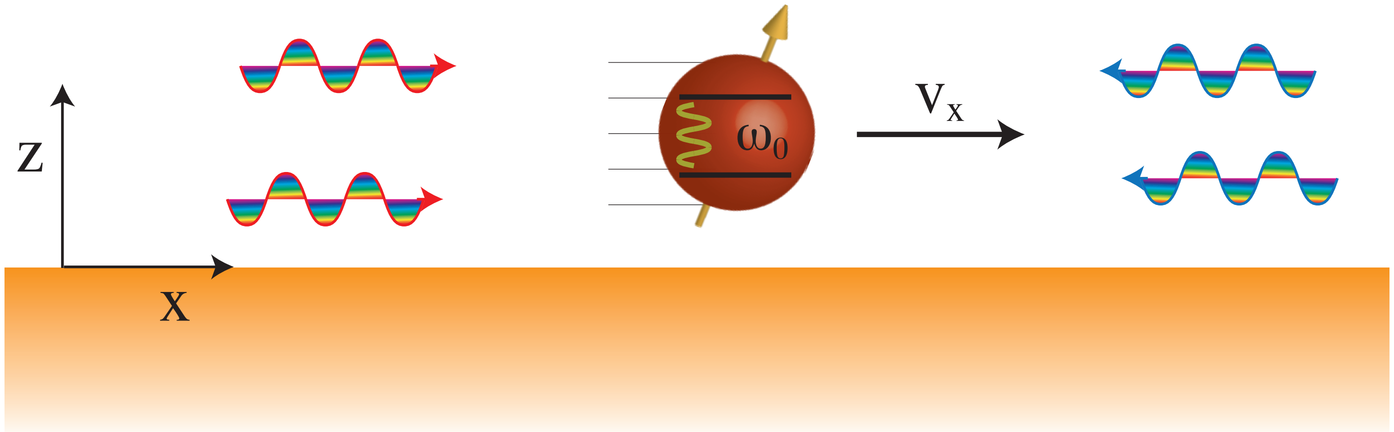
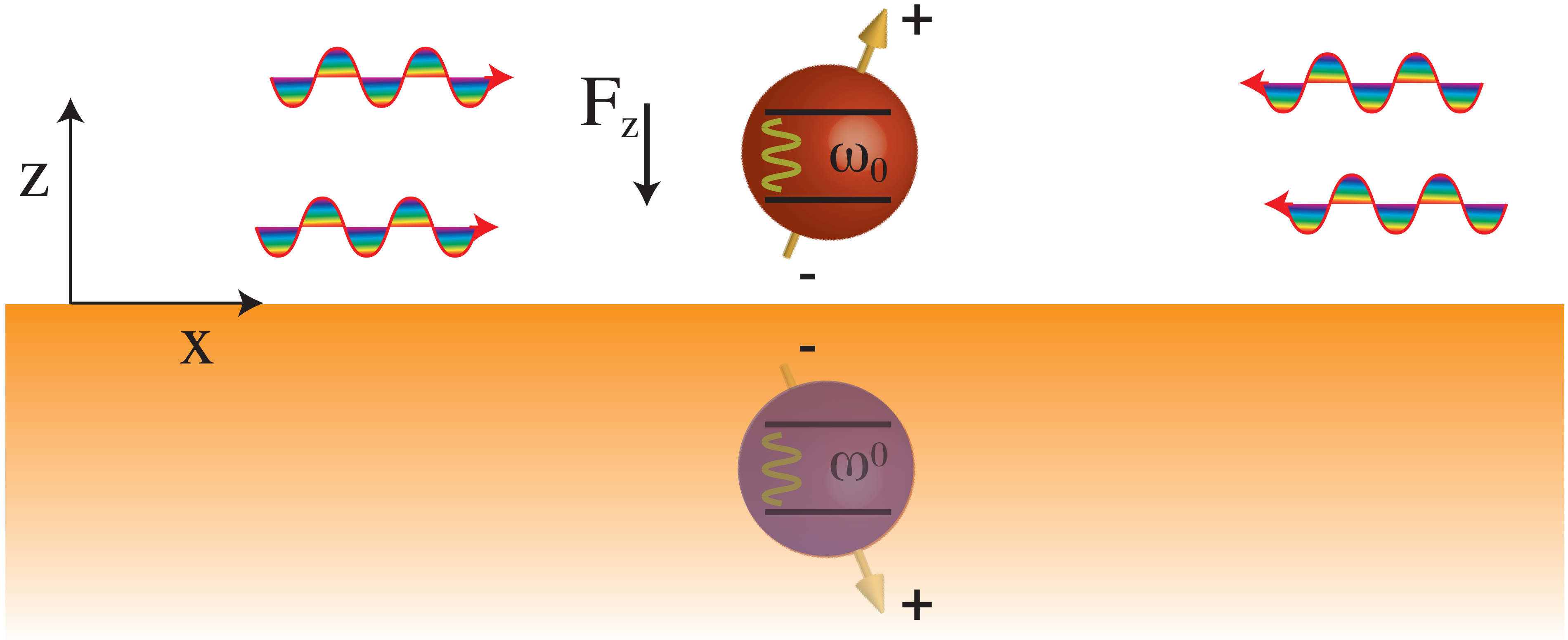
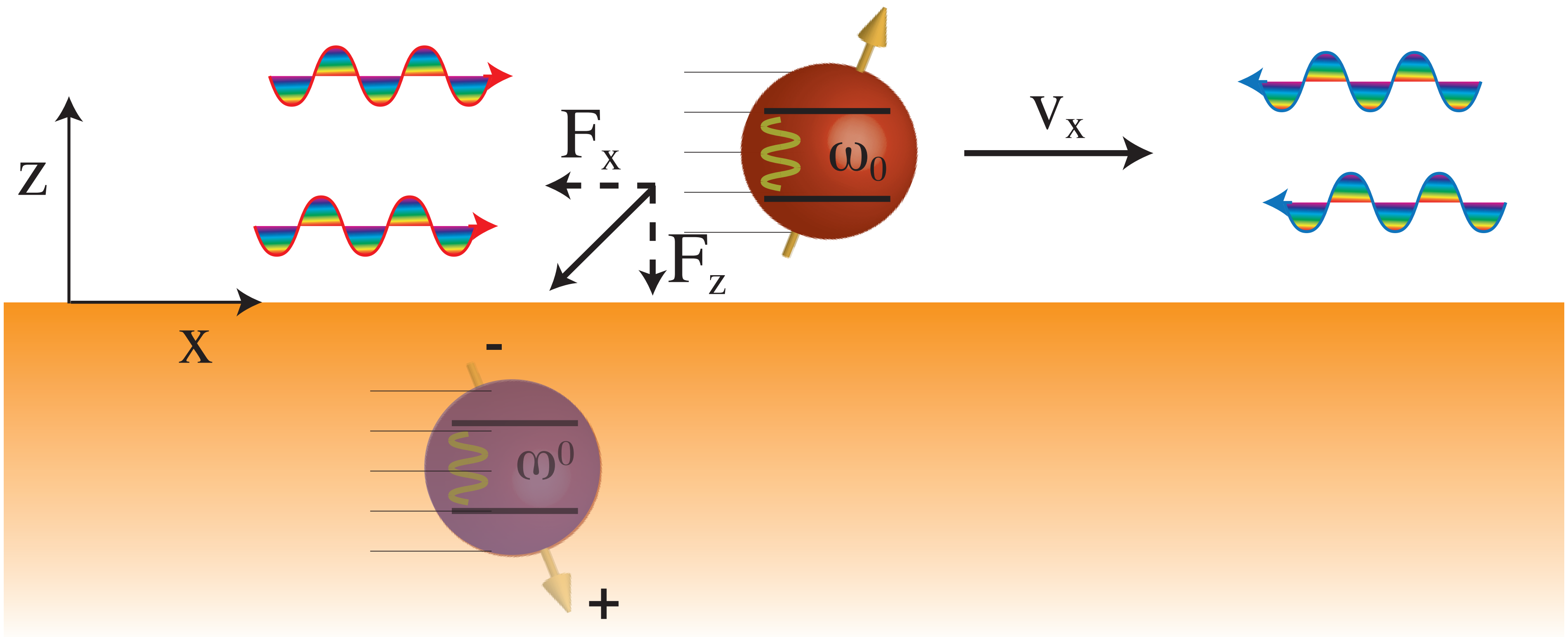


- What is quantum friction?
- Quantum friction from non-equilibrium FDT
- Quantum friction from t-dep. perturbation theory
- Key results
  - Quantum friction depends on the way the atom is boosted
  - Quantum friction is cubic in velocity

An intuitive picture

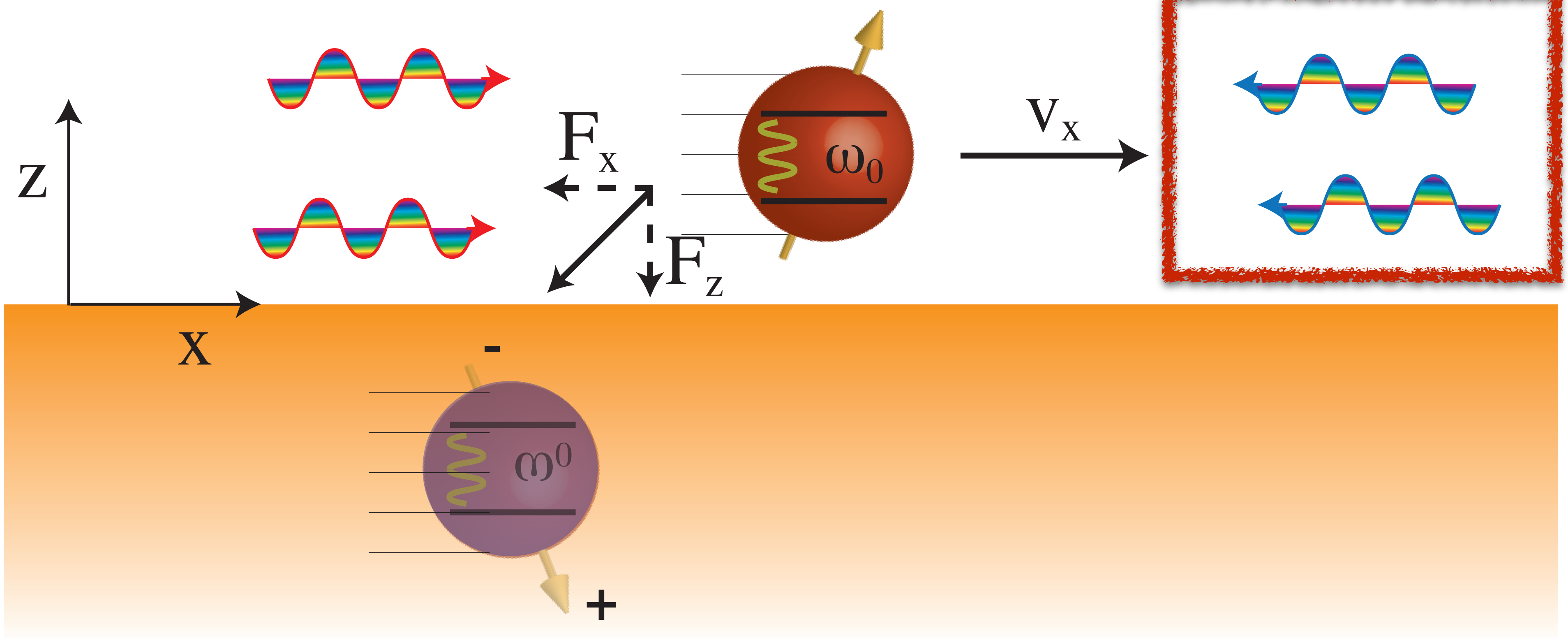


An intuitive picture



An intuitive picture

Photons and plasmon field perceived with a Doppler shifted frequency



A variety of predictions

Zero temperature atom-surface quantum friction

Authors	Low velocity dependency	Distance dependency	Comments
Mahanty 1980	v	z^{-5}	Approach similar to the calculations of vdW forces but with mistakes
Schaich and Harris 1981	v	z^{-10}	Two-state atom with a transition dipole moment normal to a metal surface
Scheel and Buhmann 2009	v	z^{-8}	Master-equation approach for **multilevel atoms** and quantum regression "theorem".
Barton 2010	v	z^{-8}	Perturbation theory using Fermi's golden rule. **Harmonic oscillator.**
Philbin and Leonhardt 2009	0	0	Relativistic calculations and analytical/numerical evaluation of the Green's tensor
Dedkov and Kyasov 2012	v^3	z^{-7}	Fluctuation-dissipation theorem applied to the dipole atom as well as to the electric field

Fluctuational electrodynamics

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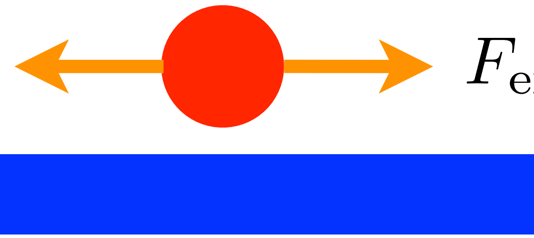
$$F_{\text{fric}}(t) = \langle \hat{\mathbf{d}}(t) \cdot \partial_x \hat{\mathbf{E}}(\mathbf{r}_a(t), t) \rangle$$



$F_{\text{ext}}(t)$

- Prescribed motion

$$\mathbf{r}_a(t) = \begin{cases} (x_0, y_a, z_a) \text{ for } t < t_a \\ (x_{\text{accel}}(t), y_a, z_a) \text{ for } t_a < t < 0 \\ (x_a + v_x t, y_a, z_a) \text{ for } t > 0 \end{cases}$$

$$m_a \ddot{x}_a(t) = F_{\text{ext}}(t) + F_x(t)$$

- Stationary $(t \to \infty)$ frictional force

$$\boxed{F_{\text{fric}} = \operatorname{Re}\left\{ \frac{2}{\pi} \int \frac{d^2\mathbf{k}}{(2\pi)^2} k_x \int_0^\infty d\omega \int_0^\infty d\tau e^{-i(\omega - k_x v_x)\tau} \operatorname{Tr}[\underline{C}(\tau; v_x) \cdot \underline{G}_I(\mathbf{k}, z_a, \omega)] \right\}}$$

$$\underline{C}_{ij}(\tau; v_x) = \operatorname{tr}\{\hat{d}_i(0) \hat{d}_j(-\tau) \hat{\rho}(\infty)\}$$

- **No general results for non-equilibrium state** $\hat{\rho}(\infty) = ???$

Non-equilibrium FDT

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- Dipole moment $\hat{\mathbf{d}} = \mathbf{d}\hat{q}$ $\quad\quad \ddot{\hat{q}}(t) + \omega_a^2 \hat{q}(t) = \frac{2\omega_a}{\hbar}\mathbf{d}\cdot\hat{\mathbf{E}}(\mathbf{r}_a(t), t)$

- **Dynamic polarizability of moving atom**

$$\underline{\alpha}_{ij}(\omega; v_x) = \frac{2\omega_a}{\hbar}\mathbf{d}_i\mathbf{d}_j\left[-\omega^2 + \omega_a^2 - \frac{2\omega_a}{\hbar}\int\frac{d^2\mathbf{k}}{(2\pi)^2}\mathbf{d}\cdot\underline{G}(\mathbf{k}, \omega + k_x v_x)\cdot\mathbf{d}\right]^{-1}$$

- **An exact, non-equilibrium fluctuation-dissipation relation**

$$\boxed{\underline{S}(\omega; v_x) = \frac{\hbar}{\pi}\theta(\omega)\underline{\alpha}_I(\omega; v_x) - \frac{\hbar}{\pi}\underline{J}(\omega; v_x)}$$

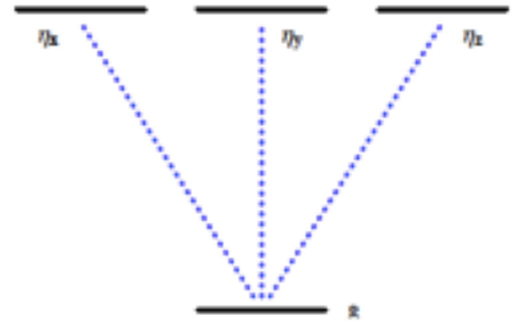
Non-equilibrium FDT in classical models have the same form

(Chetrite et al. 2008)

$$\underline{J}(\omega; v_x) = \int\frac{d^2\mathbf{k}}{(2\pi)^2}\left[\theta(\omega) - \theta(\omega + k_x v_x)\right]\underline{\alpha}(\omega; v_x)\cdot\underline{G}_I(\mathbf{k}, \omega + k_x v_x)\cdot\underline{\alpha}^*(\omega; v_x).$$

- Using $\underline{S}(\omega; v_x)$ one can obtains $\quad \boxed{F_{\text{fric}} \approx -\frac{45\hbar}{256\pi^2\epsilon_0}\alpha_I'(z_a, 0)\Delta_I'(0)\frac{v_x^3}{z_a^7}}$

Other approach: pert. theory



- Multi-level atom, initially in its ground state
- Polariton field (EM+matter) also in its ground state

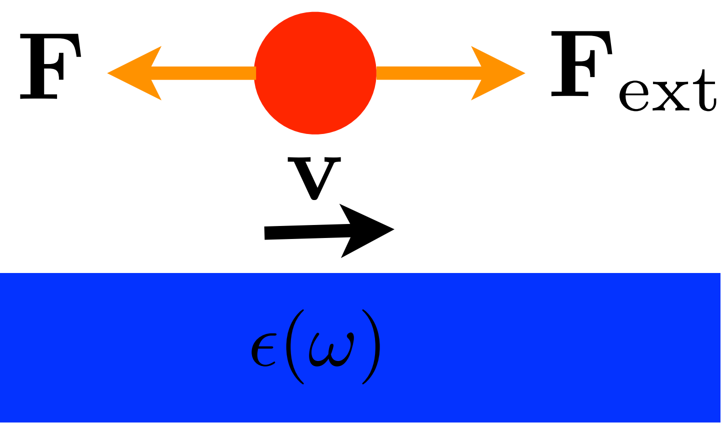
$$\rho(0) = |g\rangle\langle g| \otimes |\mathrm{vac}\rangle\langle\mathrm{vac}|$$

- Prescribed motion $\mathbf{r}_a(t) = \begin{cases} (x_0, y_a, z_a) \text{ for } t < t_a \\ (x_{\mathrm{accel}}(t), y_a, z_a) \text{ for } t_a < t < 0 \\ (x_a + v_x t, y_a, z_a) \text{ for } t > 0 \end{cases}$

- Our goal is to calculate:

radiative frictional force $\mathbf{F}$

radiative frictional power $P = -\mathbf{v} \cdot \mathbf{F}$



Transition amplitudes

- Polariton field Hamiltonian (near-field regime)

$$\hat{\Phi}(\vec{r},t) = \int \mathrm{d}^2k \int_0^\infty \mathrm{d}\omega \, (\hat{a}_{\mathbf{k}\omega}\, \phi_{\mathbf{k}\omega} \exp(\mathrm{i}\mathbf{k}\cdot\mathbf{r} - \mathrm{i}\omega t) + \mathrm{h.c.}) \qquad \phi_{\mathbf{k}\omega} = \frac{\sqrt{\omega\Gamma\omega_p^2/2}}{\omega^2 + \mathrm{i}\omega\Gamma - \omega_S^2}\sqrt{\frac{\hbar}{2\pi^2 k}}\, \mathrm{e}^{-kz}$$

- Atom-field coupling $\hat{V}(t) = -\hat{D}_i(t)\hat{E}_i(\vec{r}(t),t) = \hat{D}_i\partial_i\hat{\Phi}(\vec{r}(t),t)$

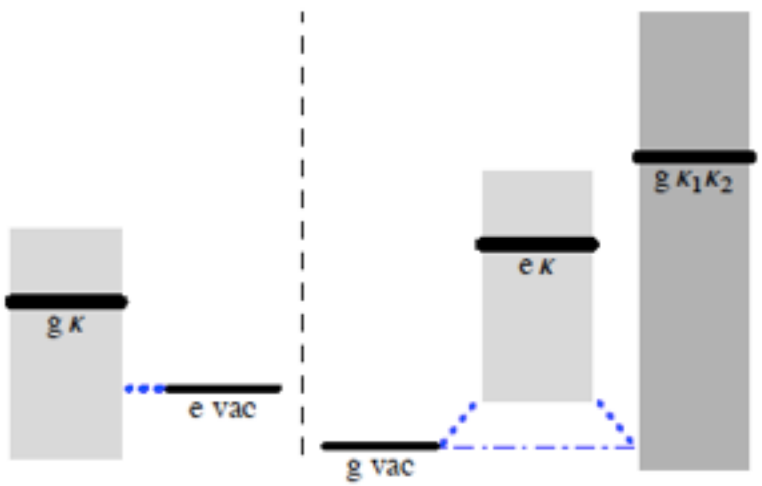
- Atom-field states

$$|\Psi(t)\rangle = (1 + c_0^{(2)}(t))|g,\mathrm{vac}\rangle + \sum_{\vec{\eta}} \int \mathrm{d}^3\kappa \, (c_1^{(1)}(t) + c_1^{(3)}(t))|\vec{\eta},\kappa\rangle$$
$$+ \frac{1}{2}\int \mathrm{d}^3\kappa_1\, \mathrm{d}^3\kappa_2\, c_2^{(2)}(t)|g,\kappa_1\kappa_2\rangle + \ldots$$

(third order expansion in powers of d)

$$\boxed{c_n^{(p)}(t)}$$

transition amplitudes for states with n-photons in the p-th pert. order



$$\begin{aligned}
\langle g,\mathrm{vac}|\hat{V}(t)|\vec{\eta},\kappa\rangle &= \mathrm{i}\, d\,(\vec{\eta}\cdot\vec{k})\phi_\kappa \exp[-\mathrm{i}(\Omega+\omega)t + \mathrm{i}\mathbf{k}\cdot\mathbf{r}(t)]\,,\\
\langle \vec{\eta},\kappa|\hat{V}(t)|g,\kappa_1\kappa_2\rangle &= \mathrm{i}\, d\,(\vec{\eta}\cdot\vec{k}_1)\phi_{\kappa_1}\, \mathrm{e}^{\mathrm{i}(\Omega-\omega_1)t + \mathrm{i}\mathbf{k}_1\cdot\mathbf{r}(t)}\delta(\kappa-\kappa_2)\\
&\quad + \mathrm{i}\, d\,(\vec{\eta}\cdot\vec{k}_2)\phi_{\kappa_2}\, \mathrm{e}^{\mathrm{i}(\Omega-\omega_2)t + \mathrm{i}\mathbf{k}_2\cdot\mathbf{r}(t)}\delta(\kappa-\kappa_1)\,,\\
\langle \vec{\eta},\mathrm{vac}|\hat{V}(t)|g,\kappa\rangle &= \mathrm{i}\, d\,(\vec{\eta}\cdot\vec{k})\phi_\kappa \exp[\mathrm{i}(\Omega-\omega)t + \mathrm{i}\mathbf{k}\cdot\mathbf{r}(t)]\,.
\end{aligned}$$

Frictional force and power

- Quantum friction force operator:

$$\hat{\mathbf{F}}(t) = \int \mathrm{d}^2 k \int_0^\infty \mathrm{d}\omega \left(\mathbf{k}(\hat{\vec{D}}(t) \cdot \vec{k}) \phi_{\mathbf{k}\omega} \hat{a}_{\mathbf{k}\omega} \exp(\mathrm{i}\mathbf{k} \cdot \mathbf{r}(t) - \mathrm{i}\omega t) + \mathrm{h.c.} \right)$$

- Radiated power:

$$P = P_1 + P_2$$

$$P_1 = \lim_{t\to\infty} \sum_{\vec{\eta}} \int \mathrm{d}^3\kappa \, \hbar(\Omega + \omega) \frac{|\langle \vec{\eta}, \kappa | \Psi(t) \rangle|^2}{t}$$

$$P_2 = \frac{1}{2} \lim_{t\to\infty} \int \mathrm{d}^3\kappa_1 \int \mathrm{d}^3\kappa_2 \, \hbar(\omega_1 + \omega_2) \frac{|\langle g, \kappa_1 \kappa_2 | \Psi(t) \rangle|^2}{t}$$

- To <u>second order</u> in perturbation theory, the force and power are exponentially small

$$\mathbf{F}^{(2)} \propto \exp(-2\Omega z / v_x)$$
$$P^{(2)} \propto \exp(-2\Omega z / v_x)$$

- To <u>fourth order</u> in perturbation theory, great care must be exercised because of possible interferences between the amplitudes $c_n^{(p)}(t)$

Ideal case: constant v always

We consider first the idealized case of the atom moving at constant velocity at all times $\mathbf{r}(t) = \mathbf{v}t, \ \forall t$

- One-photon power dissipated P_1

$$P_1^{(4)} \to 2\ \mathrm{Re}[c_1^{(1)*}(t) c_1^{(3)}(t)] \approx -\gamma_g t P_1^{(2)}$$

hence, it is exponentially suppressed

- Two-photon power dissipated P_2

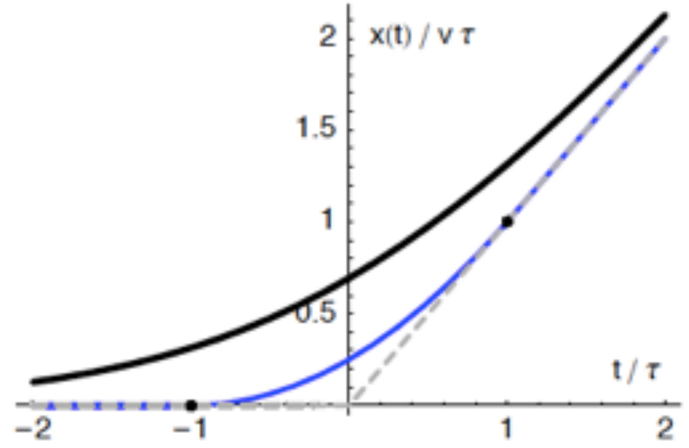
$$P_2^{(4)} \to |c_2^{(2)}|^2 \propto \frac{v_x^4}{z_a^{10}}$$

- Frictional force $F_x^{(4)} \propto \dfrac{v_x^3}{z_a^{10}}$

Boosting the atom: $\mathbf{v}(t)$

We now consider the more realistic case in which the atom is boosted from its initial rest position to a final state in which it moves at constant velocity

Curve	$\dot{x}(t)$	$\ddot{x}(t)$
thick black	$v/(1+e^{-t/\tau})$	$v\tau^{-1}/(2+2\cosh t/\tau)$
thin blue	$\begin{cases} 0 & \text{for } t<-\tau \\ (t+\tau)v/(2\tau) & \text{for } -\tau<t<\tau \\ v & \text{for } t>\tau \end{cases}$	$\begin{cases} 0 & \text{for } t<-\tau \\ v/(2\tau) & \text{for } -\tau<t<\tau \\ 0 & \text{for } t>\tau \end{cases}$
dashed gray	$\begin{cases} 0 & \text{for } t<0 \\ v & \text{for } t>0 \end{cases}$	$v\delta(t)$



- Two-photon power $P_2^{(4)} = P_A + P_B$

  $P_A \propto v_x^4$ is independent of the boost ⟵ (Barton 2010)

  $P_B \propto v_x^2\ f(\tau)$ depends on the boost, and is exponentially suppressed for adiabatic boosts $\omega\tau \gg 1$

- One-photon power $P_1^{(4)} = -P_B$ ⟵ subtle cancellation!

Hence, perturbation theory also gives cubic-in-v quantum friction

Orders of magnitude

- **Near-field quantum friction**

surface's electrical resistivity

static atomic polarizability

$$F_{\text{fric}} \approx -\frac{45\hbar\rho^2\alpha_0^2}{512\pi^3}\frac{v_x^3}{z_a^{10}}$$

- **Example:** ground state $^{87}\mathrm{Rb}$ flying over a silicon surface

$$\alpha_0 = 5.26 \times 10^{-39}\ \mathrm{Hz/(V/m)^2}$$
$$\rho = 6.4 \times 10^2\ \Omega\ \mathrm{m}$$
$$v_x = 340\ \mathrm{m/s}$$
$$z_a = 10\ \mathrm{nm}$$

$$\Rightarrow\ F_{\text{fric}} \approx -1.3 \times 10^{-20}\ \mathrm{N}$$

!

- **How to enhance it? How to measure it?**
  - excited atomic states?
  - higher velocities?
  - materials with higher resistivities?
  - macroscopic bodies?
  - ???
  - atomic interferometry?
  - near-field AFM?
  - ???

Conclusions

- Atom-surface quantum friction from general non-equilibrium stat. mech.
- Non-equilibrium FDT predicts a cubic-in-v frictional force
- Atom-surface quantum friction from t-dependent perturbation theory

  - Dependency on boost history

  - Subtle cancellation between one- and two-photon dissipated power. This results, again, in cubic-in-v quantum friction

- At high temperatures (classical limit), linear-in-v frictional force
- Same analysis possible for quantum friction between macroscopic bodies